DOCUMENT RESUME

ED 318 987 CS 009 997

AUTHOR	Gentner, Dedre; Jeziorski, Michael
TITLE	Historical Shifts in the Use of Analogy in Science. Technical Report No. 498.
INSTITUTION	Bolt, Beranek and Newman, Inc., Cambridge, Mass.; Illinois Univ., Urbana. Center for the Study of Reading.
SPONS AGENCY	Office of Educational Research and Improvement (ED), Washington, DC.; Office of Naval Research, Arlington, Va.
PUB DATE	Apr 90
CONTRACT	G0087-C1001-90; N00014-85-K-0559; NR667-551
NOTE	36p.
PUB TYPE	Reports - Evaluati /Feasibility (142) -- Historical Materials (060)
EDRS PRICE	MF01/PC02 Plus Postage.
DESCRIPTORS	Cognitive Style; *Creative Thinking; *Science History; Scientific Methodology; Scientists; Thinking Skills
IDENTIFIERS	*Analogical Reasoning; *Analogy; Theory Development

ABSTRACT

Noting that analogy is widely considered to be an important mechanism of scientific thinking and a source of creative insight in theory development, this report considers the implicit constraints that determine analogical soundness. The report first examines the constraints that govern analogical reasoning as it is predicated currently. The report then traces the scientific uses of analogy through three time periods and contrasts the styles of analogizing practice by scientists at different points in history. The report concludes that the notion of analogical soundness has evolved over time. (Four tables of data, four figures, and 18 footnotes are included.) (RS)

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April 1990

Center for the Study of Reading

TECHNICAL REPORTS

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
174 Children's Research Center
51 Gerty Drive
Champaign, Illinois 61820

CENTER FOR THE STUDY OF READING

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The work upon which this publication was based was supported in part by the Office of Naval Research under Contract No. N00014-85-K-0559, NR667-551, and in part by the Office of Educational Research and Improvement under Cooperative Agreement No. G0087-C1001-90 with the Reading Research and Education Center. The publication does not necessarily reflect the views of the agencies supporting the research.

Abstract

Analogy is widely considered to be an important mechanism of scientific thinking and a source of creative insight in theory development. In this report we consider the implicit constraints that determine analogical soundness. We first examine the constraints that govern analogical reasoning as it is predicted today. We then trace the scientific uses of analogy through three time periods and contrast the styles of analogizing practice by scientists at different points in history. This comparison suggests that the notion of analogical soundness has evolved over time.

HISTORICAL SHIFTS IN THE USE OF ANALOGY IN SCIENCE

Analogy is widely considered to be an important mechanism of scientific thinking and a source of creative insight in theory development (e.g., Tweney, 1989). No less an authority than Johannes Kepler stated: "And I cherish more than anything the Analogies, my most trustworthy masters. They know all the secrets of Nature, and they ought to be least neglected in Geometry" (quoted in Polya, 1954, p. 12). In addition to its uses in scientific discovery, analogy functions as part of the workaday tool kit of science. In instruction, novices are told to think of electricity as analogous to water or of addition as analogous to piling up blocks, and in problem-solving analogy is a standard tool among both experts and novices (e.g., see Clement, 1981; Collins & Gentner, 1987; Gentner & Gentner, 1983; Van Lehn & Brown, 1980). Finally analogy is also used in everyday reasoning, as when the stock market is said to "climb to dizzying heights" or when there is said to be a "balance of trade" (see Lakoff & Johnson, 1980).

Yet for all its usefulness, analogy is never formally taught to us. We seem to think of analogy as a natural human skill, and of the practice of analogy in science as a straightforward extension of its use in common-sense reasoning. For example, William James believed that "men, taken historically, reason by analogy long before they have learned to reason by abstract characters" (James, 1890, *II*, p. 363). All this points to an appealing intuition: that a facility for analogical reasoning is an innate part of human cognition, and that the concept of a sound analogy is universal.

In this report we question this intuition. We begin by discussing a framework for analogical reasoning. We then present examples of scientific uses of analogy from three time periods, working backward from Sadi Carnot (1796-1832) to Robert Boyle (1627-1691) and finally to a set of alchemists active before 1550.[1] On the basis of these examples, we contrast the style of analogizing practiced by scientists at different points in history. We believe there are significant differences in the style of thinking, in what was felt to constitute rigor, in what was accepted as a sound argument and a justifiable conclusion--in short, in what has been taken to be the logical and scientific use of analogical reasoning. This raises questions as to whether the standards of analogical rigor are universal and innate, or whether they are instead culturally and historically defined.

Before we present our historical analyses, we need to make explicit the constraints that govern analogical reasoning as it is practiced today. We will then be in a position to compare the uses of analogy across history.

A Framework for Interpreting and Evaluating Analogy

Analogy can be viewed as a kind of similarity, but not all similarity is analogy. Indeed, analogy gains much of its power from the selectivity of the commonalities it suggests. When processing an analogy, people focus on certain kinds of commonalities and ignore others. For example, imagine a bright student reading the analogy "A cell is like a factory." It is unlikely that he or she would decide that cells are made of brick and steel and have smokestacks. Instead the student would probably realize that, like a factory, a cell must take in available resources to keep itself operating and to generate its products. This focus on abstract commonalities is what makes analogy so illuminating. In the next section, we present a way of clarifying this intuition.

Structure-Mapping and Ideal Analogical Competence

The theoretical framework for this research is the structure-mapping theory of analogy (Gentner, 1980, 1983, 1988, 1989). This theory aims to describe the implicit constraints that characterize modern analogical aesthetics. The basic intuition is that an analogy is a mapping of knowledge from one domain (the base) into another (the target), which conveys that a system of relations that holds among the base objects also holds among the target objects. Thus an analogy is a way of noticing relational commonalities independently of the objects in which those relations are embedded. In interpreting an analogy, people seek to put the objects of the base in one-to-one correspondence with the objects of the target so as to obtain maximum structural match. The corresponding objects in the base and target do not have to resemble each other at all; object correspondences are determined by roles in the matching relational structures. Central to the mapping process is the principle of systematicity: In selecting among possible matching relations, people prefer interconnected systems; that is, they prefer sets of predicates linked by higher order relations such as CAUSE or IMPLIES, rather than isolated predicates. The systematicity principle is a structural expression of our tacit preference for coherence and deductive power in interpreting analogy.

Besides analogy, other kinds of similarity matches can be distinguished in this framework, according to whether the match is one of relational structure, object descriptions, or both. Recall that *analogies* discard object descriptions and map relational structure. *Mere-appearance* matches are the opposite: They map aspects of object descriptions and discard relational structure. *Literal-similarity* matches map both relational structure and object-descriptions.

As an example, consider the Rutherford analogy between the solar system and the hydrogen atom. Imagine a person hearing it for the first time. (Assume some prior knowledge about the solar system.) The person must [2]

Set up the object correspondences between the two domains: sun → nucleus and planet → electron.

Discard object attributes, such as YELLOW (sun).

Map base relations such as MORE MASSIVE THAN (sun, planet) to the corresponding objects in the target domain.

Observe systematicity, that is, seek a system of interconnected relations such as MORE MASSIVE THAN (sun, planet) and REVOLVES-AROUND (planet, sun) that are linked by higher order constraining relations, such as CAUSE, such that the whole system can apply in the target as well as the base. Here, the deepest potentially common system of relations--at least in 1906--is the central-force system:

CAUSE {AND [ATTRACTS (sun, planet)],
[MORE-MASSIVE-THAN (sun, planet)],
REVOLVE-AROUND (planet, sun)}.

Discard isolated relations, such as HOTTER THAN (sun, planet).

Systematicity

Central to our understanding about analogy is that it conveys a system of connected knowledge, not a mere assortment of independent facts. The systematicity principle is included to formalize this tacit preference for coherence and deductive power in analogy. The *systematicity principle* states that in analogy there is an implicit selection rule to seek a common system of relations (i.e., a system from the base that can also apply in the target). That is, among the possible commonalities between base and target, we seek to find an interconnected predicate structure in which higher order predicates enforce constraints among lower order predicates.[3] A predicate that belongs to such a system is more likely to be included in the analogy than is an isolated predicate. By promoting deep relational chains, the systematicity principle operates to promote predicates that participate in causal chains and other constraining relations.[4]

The structure-mapping principles have received convergent theoretical support in artificial intelligence and psychology, as well as in other areas of cognitive science (Burstein, 1983; Hesse, 1966; Hofstadter, 1981; Indurkhya, 1985; Reed, 1987; Rumelhart & Norman, 1981; Winston, 1980, 1981, 1982). There is widespread agreement on the basic elements of one-to-one mappings of objects with carry-over of predicates Further, many of these researchers use the systematicity principle, or a close relation, as their selection principle. There is also empirical support for the psychological predictions of structure-mapping theory (Gentner, 1980, 1989; Gentner & Gentner, 1983; Gentner & Toupin, 1986; Reed, 1987;

Schumacher & Gentner, 1987). In particular, there is evidence to suggest that adults do indeed observe the aesthetic rules of rigor that structure mapping suggests: that is, that they focus on shared systematic relational structure in interpreting analogy. First, adults tend to include relations and omit attributes in their interpretations of analogy; and second, adults judge analogies as more apt and more sound if they share systematic relational structure (Clement & Gentner, 1988; Gentner, 1989; Gentner & Clement, 1988; Gentner & Landers, 1985; Gentner & Rattermann, in preparation).

The Rules of Analogical Rigor

On the basis of the foregoing discussion, we propose a set of five implicit rules that modern scientists use in analogical reasoning. The first three rules, based directly on structure mapping, state constraints internal to a particular interpretation; the last two rules state external constraints:

1. Structural consistency is maintained. This means, first, that objects are placed in consistent one-to-one correspondence; that is, a given object in one domain cannot have more than one counterpart in the analogous domain. Multiple mappings diminish the clarity of the match. We will refer to violations of this principle as n - 1/1 - n mappings. Second, the connectivity among predicate structures is maintained. When two predicates are placed in correspondence, the elements that support them (i.e., that are their arguments) must also be placed in correspondence.

2. Attributes are discarded, whereas relations are preserved. The focus of the analogy is on matching systems of relations, not objects and their surface attributes. We do not care whether, for example, the nucleus resembles the sun as an object, only whether it participates in the same system of relations.

3. The systematicity principle is used to select the most informative common relational network. Lower order relations that are not contained within such a network are discarded. Thus, in the Rutherford analogy, the lower order relation HOTTER-THAN (sun, planet) is not part of the analogy because, although it participates in a systematic relational structure in the base (that of heat transfer), that system is not shared with the target.

4. Between-domain relations do not strengthen the analogy. Only commonalities improve the match; additional associations between the two domains are irrelevant to the soundness of the match. For example, in the analogy between the solar system and the atom, it does not make the analogy more sound to observe that the solar system is *made up of* atoms.

5. Mixed analogies are avoided. An analogy that builds a relational network in the target domain by selecting isolated relations from several base domains is not considered sound. The relational network to be mapped should be entirely contained within one base domain.

In discussing this last "no mixed analogies" rule, we must distinguish mixed analogies from allowable cases of multiple analogies (Burstein, 1983; Collins & Gentner, 1987; Schumacher & Gentner, 1987; Spiro, Feltovich, Coulson, & Anderson, 1989). In some cases, several parallel base analogues are used to make the same point concerning the target domain. Here, although several analogies embody the same abstraction, each mapping stands on its own independently of the others (see the discussion of Boyle's analogies, below). Another allowable case is that in which the target can be partitioned into separate subsystems, each with a different base analogue. A third allowable case of multiple analogies is that in which the analogies are alternatives, each used to illuminate a different aspect of the target (e.g., electricity as flowing water or as crowds of moving particles [Gentner & Gentner, 1983] or variables viewed as containers or as unknowns [Burstein, 1983]). It does not entail a loss of rigor if different analogies are each used separately and consistently. However, when different analogies are merged, there is often a loss of precision, since the various analogues may suggest different object correspondences. A reasoner who shifts among analogies without establishing firm rules of intersection risks a lack of clarity in his or her conclusion. Thus, whereas *multiple* analogies for the same domain are sometimes perfectly rigorous, *mixed* analogies violate the consensual rules of sound thinking and are vulnerable to challenge.

Finally, analogy between domains is a separate issue from causation between domains. Although analogy can be used to infer that identical causal relations exist *within* one domain as within the other, it cannot be used to infer causation *between* the base and target domains; nor does evidence of a causal relation between the domains strengthen an analogy.[5]

Table 1 summarizes these rules of soundness. Note that although the rules concern only the soundness evaluation, they are intimately related to the process of making new inferences. As mentioned above, new inferences are typically made by a process of *system completion* after some degree of match has been established. The most typical kind of *candidate inference* occurs when a predicate is found such that (a) it exists in the base but not in the target; (b) it belongs to an interconnected system of predicates in the base; and (c) other predicates in its system have matching predicates in the target. Then the predicate is postulated to exist in the target as well. That is, the partially matching system is completed in the target.

[Insert Table 1 about here.]

The five rules do not tell us whether the analogy is factually true; they tell us only whether it is sound. Verifying the factual validity of an analogy is a separate process. Soundness rules are enormously helpful in this process, however, because they tell us what must be true in the target in order for the analogy to be valid. In a rigorous system of matches, even one significant disconfirmation can invalidate a whole analogy. Thus soundness and validity go hand in hand in hypothesis generation and testing.

In modern cognitive aesthetics, the soundness of an analogy rests solely on the systematic struetural match between the two domains. Given these modern rules of analogical rigor, we now turn to the question of whether scientists have always adhered to these principles. We begin with Carnot, the most recent example, and move in reverse chronological order.

Historical Uses of Analogy

Sadi Carnot

The French scientist Sadi Carnot (1796-1832) is well known as a pioneer of modern thermodynamics. He described the Carnot cycle for heat engines that is still taught as an ideal energy conversion system, and he laid the foundation for the later discovery of the equivalence of heat and work. In his treatise on heat, Carnot presented a powerful analogy between heat and water that clarified his position and generated new questions. His use of analogy is prototypical of the rules of rigor described above, and can stand as an example of the modern use of analogy.

Before explaining Carnot's analogy, we present a short summary of his work. In 1824, Carnot published *Reflexions sur la puissance motrice du feu* (Reflections on the Motive Power of Fire). In this book, he describes the functioning of a hypothetical engine that can convert heat energy to work. This engine consists of a cylinder filled with gas and fitted with a frictionless piston that can move freely inside the cylinder. During a four-stage cycle, the gas inside is expanded by contact with a heat source (isothermal expansion) and allowed to continue dilation after the source is removed (adiabatic expansion). The gas is then compressed by transmission of heat to a colder body (isothermal compression), and the volume further decreases after removal of the cold body (adiabatic compression), restoring the original conditions of the system. The point of this exercise is that the engine will have absorbed a certain amount of heat and converted it to mechanical work through the movement of the piston. The operation of such an ideal engine became known as the Carnot cycle, and was an important contribution to the early development of thermodynamics.

Early in his *Reflexions*, Carnot introduces the analogy between water falling through a waterfall and caloric (heat) falling through a heat engine. The basic notion of an analogy between heat and fluid was not new. Indeed, the dominant theory of heat at the time was the *caloric* theory,[6] which defined heat as a weightless fluid that shared certain properties of ordinary matter. Like other matter, caloric was a conserved quantity, incapable of being created or destroyed. Thus the idea of some commonality between heat and water was not new with Carnot, since both are instantiations of a common abstraction (i.e., both are fluids). What was new was the thoroughness of his development of the analogy--the extent to which explicit causal structures from the water domain were applied in the heat domain.

Carnot uses the analogy to set forth the principles of a heat engine, and then derives further insights about the motive power of a heat engine by analyzing the system of relations in the water engine.[7]

> [1] According to established principles at the present time, we can compare with sufficient accuracy the motive power of heat to that of a waterfall. Each has a maximum that we cannot exceed, whatever may be, on the one hand, the machine which is acted upon by the water, and whatever, on the other hand, the substance acted upon the heat.
>
> [2] The motive power of a waterfall depends on its height and on the quantity of the liquid; the motive power of heat depends also on the quantity of caloric used, and on what may be termed, on what in fact we will call, the *height of its fall*, that is to say, the difference of temperature of the bodies between the higher and lower reservoirs.
>
> [3] In the waterfall the motive power is exactly proportional to the difference of level between the higher and lower reservoirs. In the fall of caloric the motive power undoubtedly increases with the difference of temperature between the warm and the cold bodies; but we do not know whether it is proportional to this difference. We do not know, for example, whether the fall of caloric from 100 to 50 degrees furnishes more or less motive power than the fall of this same caloric from 50 to zero. It is a question which we propose to examine hereafter.
>
> (Carnot, 1977, p. 15; numbers and paragraph breaks are inserted for convenience; the original passage is continuous.)

In [1], Carnot introduces the analogy between the motive power of heat and the motive power of water and establishes a simple, yet important parallel: Just as the amount of power produced by a given fall of water is limited, the power attainable from a certain transfer of heat is limited. This section establishes a set of correspondences between the elements of the heat system and the elements of the water system, as shown in Figures 1 and 2.

[Insert Figures 1 and 2 about here.]

In [2], Carnot explicates the analogy more explicitly by comparing the difference in temperature between two bodies to the height of the fall in a waterfall.[8] This correspondence between difference in temperature of two bodies and difference in levels of two reservoirs is crucial to the analogy. Carnot uses this correspondence in a proposed higher order relation; he asserts that, in each case, the power produced by the system depends on both the amount of the substance (water or caloric) that "falls" and the distance of the "drop" between levels:

> DEPENDS-ON {POWER (high, low),
> AND [DIFFERENCE (level<high>, level<low>)],
> [amount<water>]}
> →
> DEPENDS-ON {POWER (hot, cold),
> AND [DIFFERENCE (temperature<hot>, temperature<cold>)],
> [amount<heat>]}

This combination of inferences--the fact that power depends on both the difference in level and the amount of "substance" involved--solidifies the analogy between the two engines. Figure 1 shows the common relational structure that holds for water and heat; Figure 2 sets forth the predicates in the water domain that belong to the analogy.

In [3], Carnot demonstrates the use of analogy in suggesting new hypotheses. He notes a higher order relation in the domain of water power (the fact that the power produced by a given fall of water is directly proportional to the difference between levels). He then questions whether the same relation exists for heat engines; that is, does the power produced by a given "fall" of caloric remain constant, regardless of the temperature at which that fall takes place? This illustrates how analogy can lead to new research hypotheses.[9]

Carnot's description and application of his analogy meet the five rules of rigorous analogical reasoning given in Table 1. Carnot pairs the objects in the two domains in one-to-one correspondence based on relational matches. He disregards attribute matches: He is not concerned with whether corresponding components share surface qualities. Rather, he focuses on common systematic relational structure. He seeks to explicate the higher order dependencies common to the two domains and to analyze the implications of these relational commonalities. Between-domain relations, such as "water contains heat," are avoided, and there is no suggestion of a mixed analogy. It is evident that the analogy was useful in revealing unresolved areas for further research. In short, Carnot's use of analogy is indistinguishable from the modern scientific use of analogy.

Robert Boyle

We now move back another 130 years to the English scientist Robert Boyle (1627-91). Boyle, considered by many to be the father of modern chemistry, was one of the first experimenters to dismiss the widespread practice of attributing human qualities such as "love" and "hate" to inanimate matter. Probably his most influential work was the *Sceptical Chymist*; appearing anonymously in 1661 and again in 1679 with additions, it "did more than any other work of the century to arouse a truly critical spirit of scientific logic in chemical thinking" (Stillman, 1924, p. 395). Among his accomplishments were a critique of the view that matter is composed of three or four principles and a proposed empirical route to discovering the number of elements, a clarification to the account of acids and alkalies, and important contributions to the understanding of the physics of gases. Boyle was a prolific writer, interested in philosophy and religion as well as the sciences, and he wrote for the layperson as well as for the scientist. He was also a prolific analogizer. He often put forth several examples or analogies for each principle he wanted to prove. These analogies seem to have been intended both as communication devices and as models to support reasoning.

A characteristic example of Boyle's use of analogy occurs in his book, *Of the great effects of even languid and unheeded local motion*, published in 1690. His purpose in this book was to demonstrate the importance of "local motion," the motions of many tiny particles. Boyle wanted to establish that the combined effects of the motion of many tiny particles--each invisible and insignificant in itself--can cause large-scale changes. He saw such effects as a unifying principle across domains such as light, sound, fire, and fluids. Although some of his points now seem to need no defense, this was not the case in his time, and he clearly felt the need to present ample evidence for this conjecture. He cites examples from one domain after another to support his claims.

Boyle's examples appear to function in two ways. First, they serve as instances of local motion and its effects--that is, as instances of a principle that can be effectively applied to several domains. The more numerous and varied the instances, the more faith we can presumably have in the principle. Second, the examples serve as analogies that can be compared to one another to yield a common structural abstraction. By comparing separate instances of local motion, Boyle led his reader to focus on the common causal system. The following excerpt illustrates his style of analogizing:

(Chap. IV) Observat. III. *Men undervalue the motions of bodies too small to be visible or sensible, notwithstanding their Numerousness, which inables them to act in Swarms.*

[1] [Boyle grants that most people think of small particles as like grains of dust, which, although invisible, cannot penetrate the bodies they fall upon. As a result, these grains cannot affect the larger bodies.]

But we may have other thoughts, if we well consider, that the Corpuscles we speak of, are, by their minuteness, assisted, and oftentimes by their figure inabled, to pierce into the innermost recesses of the body they invade, and distribute themselves to all, or at least to multitudes of the minute parts, whereof that body consists. For this being granted, though we suppose each single *effluvium* or particle to be very minute; yet, since we may suppose, even solid bodies to be made up of particles that are so too, and the number of invading particles to be not much inferior to that of the invaded ones, or at least to be exceedingly great, it not need seem incredible, that a multitude of little Corpuscles in motion (whose motion, may, for ought we know, be very swift) should be able to have a considerable operation upon particles either quiescent, or that have a motion too slow to be perceptible by sense. Which may perhaps be the better conceived by the help of this gross example:

[2] *Example of the anthill*
If you turn an Ant-hill well stocked with Ants-eggs, upside down, you may sometimes see such a heap of eggs mingled with the loose earth, as a few of those Insects, if they were yoaked together, would not be able at once to draw after them; but if good numbers of them disperse themselves and range up and down, and each lay hold of her own egge, and hurry it away, 'tis somewhat surprizing to see (as I have with pleasure done) how quickly the heap of eggs will be displaced, when almost every little egge has one of those little Insects to deal with it.

[3] *Example of wind in trees*
And in those cases, wherein the invading fluid does not quite disjoin and carry off any great number of the parts of the body it invades, its operation may be illustrated by that of the wind upon a tree in *Autumn*: for, it finds or makes it self multitudes of passages, for the most part crooked, not onely between the branches and twigs, but the leaves and fruits, and in its passing from the one side to the other of the tree, it does not onely variously bend the more flexible boughs and twigs, and perhaps make them grate upon one another, but it breaks off some of the stalks of the fruit, and makes them fall to the ground, and withall carries off divers of the leaves, that grew the least firmly on, and in its passage does by its differing act upon a multitude of leaves all at once, and variously alters their situation.

[4] *Examples of sugar and amber dissolving* [omitted here].

[5] *Examples of mercury compound dissolving* [omitted here].

[6] *Example of flame invading metal*
But to give instances in Fluid bodies, (which I suppose you will think for the more difficult part of my task,) though you will easily grant, that the flame of Spirit of wine, that will burn all away, is but a visible aggregate of such *Effluvia* swiftly agitated, as without any sensible Heat would of themselves invisibly exhale away; yet, if you be pleased to hold the blade of a knife, or a thin plate of Copper, but for a very few minutes, in the flame of pure Spirit of wine, you will quickly be able to discern by the great Heat, that is, the various and vehement agitation of the minute Corpuscles of the metal, what a number of them must have been fiercely agitated by the pervasion of the igneous particles, if we suppose, (what is highly probable,) that they did materially penetrate into the innermost parts of the metal; and whether we suppose this or no, it will, by our experiment, appear, that so fluid and yielding a body, as the flame of Spirit of wine, is able, almost in a trice, to act very powerfully upon the hardest metalls.

[7] *Example of animal spirits moving animals* [omitted here].

[8] *Example of rope contracting from humidity* [omitted here].

(Boyle, 1690, pp. 27-35)

Boyle begins by noting that laypeople may find it implausible that local motion could have large-scale effects. Laypersons, he observes, consider such motion similar to the ineffectual motion of dust in air. By analogy with dust, if particles are very small, then although they can be moved easily, their movements are inconsequential. The reason, he says, is that they do not penetrate other bodies and therefore cannot affect those bodies. Having laid out the starting intuition--that local motion is ineffective--Boyle then defends the opposite position by differentiating the analogy further. He suggests that there are some kinds of particles involved in local motion that are so small that, unlike dust particles, they can diffuse through solid objects, and that it is this penetration that allows them to create large effects. He then proceeds to present instances of this kind of local motion.

The first positive instance [2] considered by Boyle is characteristic of true analogy. He compares the ability of small particles to move large masses to that of ants to move their eggs. Although the ants are smaller than the mass of eggs, the ability of each ant to move one egg means (given appropriate relative numbers of ants and eggs) that the entire mass of eggs can be displaced by the ants. This exemplifies the principle that a large mass can be moved by the actions of many small particles. The juxtaposition of disparate examples makes it obvious that the relevant commonalities here are the relations between the objects, as shown in Figure 3; characteristics of objects are discarded. Boyle uses the anthill analogy as a rigorous structure-mapping. He does not suggest that the corpuscles involved in local motion are *like* ants in themselves; for example, he does not suggest that they are living organisms nor that they possess any instinctive notions. Nor does Boyle imply that particles of matter are white or soft or otherwise egglike. Rather, he focuses on the relational commonality: namely, that very large numbers can compensate for a very great size disadvantage, provided that penetration of the larger by the smaller can occur. Under these circumstances, many small bodies in motion can carry off a much larger body.

[Insert Figure 3 about here.]

The remaining sections provide several additional analogous examples of the effects of local motion. For example, in [3], he cites the example of wind passing through a tree, blowing off leaves and breaking branches. Similarly, in [6], Boyle presents the effects of fire on a knife blade as an instance of local motion. He perceives fire as composed of many small particles and explains the melting of metal in terms of the invasion of igneous particles into the metal, with the result that the corpuscles of metal themselves become "fiercely agitated" and the blade softens. The remaining two paragraphs, which describe "animal spirits" and the contraction of rope, respectively, make analogous points. Boyle observes that although animal spirits may be minute enough to be invisible, they are capable of propelling large animals such as elephants. He describes seeing hemp shrink in moist weather, and states that the "aqueous and other humid particles, swimming in the air, entering the pores of the hemp in great numbers, were able to make it shrink, though a weight of fifty, sixty or even more pounds of lead were tied at the end to hinder its contraction. . . . " Table 2 shows the correspondences across Boyle's set of examples.

[Insert Table 2 about here.]

A striking feature of Boyle's writing is the rapid succession of analogies he uses. Unlike Carnot, Boyle does not dwell on one pair of examples, carefully explicating the critical common relational structure. His rhetorical approach is to present his hypothesis and then provide a varied series of instances designed to demonstrate its validity. (Of course, given current domain knowledge, not all the comparisons are equally convincing.) The implicit message is that if all of these phenomena occur, the model that summarizes them must be plausible. Each paragraph contains an instance of local motion, or contrasts situations in which the principles do and do not apply. There is little surface continuity between these examples; they relate to one another by virtue of their common abstractions. The common intent is that the examples can be compared with one another to reveal in abstract model of local motion.

Boyle's use of analogy conforms to the modern standards shown in Table 1. In each of his analogies, the objects are placed in one-to-one correspondence. Object attributes are discarded: As the comparison with ants reveals, we are not intended to map the specific characteristics of the base objects into the target domain of local motion. Indeed, the sheer variety of the examples virtually guarantees that any specific object characteristics will cancel out. The analogies, in the modern tradition, are about common relational systems. The complexity of the analogies is not great--they are not as deep as Carnot's, for example--but this may be due in part to the lesser depth of knowledge of the topic area. At this early stage, Boyle simply wished to establish that the motion of many small particles can combine to produce powerful visible effects and that the condition under which this can occur is that the smaller particles be able to penetrate the larger matter. This systematic set of relations is maintained throughout these examples. Finally, in spite of the large number of examples, there are no mixed analogies nor between-domain relations; each example stands on its own as a separate instantiation of the relational structure.

Carnot and Boyle: A Summary

Boyle and Carnot differ somewhat in their use of analogy. Carnot used one analogy, explaining it precisely and then going on to use the principles in further inferencing. Boyle, in contrast, offers a whole family of analogies, one after the other. This difference may have been due to the greater depth of domain knowledge that existed in Carnot's time, or perhaps in part to a difference in their intellectual traditions.[10] Yet despite these differences, Boyle and Carnot both observe the constraints of structural consistency and systematicity. They are both essentially modern in their view of what constitutes a sound analogy.

The Alchemists

We have moved back in time from Carnot (1796-1832) to Boyle (1627-1691). So far, the analogies we have considered conform to our concept of a valid use of analogy. Now we move back to the alchemists, and analyze the forms of similarity they used in making their predictions. Rather than focusing on a single alchemist, we will consider patterns of analogizing from across the field.

The practice of alchemy, which existed in one form or another from at least A.D. 500 (Burckhardt, 1967), was a dominant force in Western scientific thought through the middle of the seventeenth century (Taylor, 1949). Although alchemy has often been maligned, it had many features that should command respect. It was based upon the belief that all matter had one origin, from which different forms had evolved. These forms were only the outer manifestations of the common "soul." They were mutable, so that substances could be converted into one another. The goal of many alchemists was to verify this theory by converting base metals such as lead into gold or silver, with the help of a putative catalyst known as the Philosopher's Stone (Redgrove, 1922).

Alchemy took as its domain the spiritual world as well as the physical world. Its adherents relied heavily on analogies between the spiritual and material planes in deriving their hypotheses. A central belief was that the "purification" of the base metals into gold was analogous to the spiritual purification of man. The resolution of either of these problems would lead to an understanding of the other (Redgrove, 1922). This "macrocosm-microcosm" analogy was a foundation of alchemical thought (Debus & Multhauf, 1966), so that "some men pursued the renewal and glorification of matter, guiding themselves by this analogy, others the renewal and glorification of man, using the same analogy" (Taylor, 1949, p. 144). The macrocosm-microcosm analogy was central to a wide network of correspondences, in which nearly every substance or procedure considered essential to the alchemist's craft had one or more analogues. These analogues could overlap. For instance, whereas metals symbolized heavenly objects (Burckhardt, 1967), a combination of two metals could be viewed as a marriage (Taylor, 1949). The alchemists exhibited prolific use of analogy when compared with earlier or later scientists. But the matches they generated were not necessarily similar to analogies we would use. Indeed, Redgrove, writing in 1922 (p. xii), stated: "The alchemists cast their theories in a mould entirely fantastic, even ridiculous--they drew unwarrantable analogies--and hence their views cannot be accepted in these days of modern science."

What were the rules that governed the alchemists' use of analogy? We begin with a prominent family of analogies that used as the base domain the egg or the seed, and as the target domain either (or both) the principles of matter or the components of a human being.

Before considering the analogies themselves, we need to give a brief historical summary of the alchemists's notions of the principles of matter.[11] Based on the works of Plato and Aristotle, alchemical thought postulated that there was a primordial source of all earthly matter called First Matter.[12] This First Matter was manifested in a small number of primary elements--fire, air, water, and earth--each of which combined two of the primary qualities--hot, cold, wet, and dry. For example, as shown in Table 3, fire was hot and dry, earth was cold and wet, etc. Transmutations occurred if the proportions of the qualities changed: For example, fire (hot and dry) could be changed into earth (specifically, into ash) by losing its heat. The alchemists were particularly interested in transmutations of metals, especially the transmutation of base metals into gold. Such a purifying transmutation would not only promise great wealth, but convincingly demonstrate that the art was true. Therefore, the theory of metals held particular interest. During the twelfth and thirteenth centuries, metals were generally held to consist of two components: mercury, which was fiery, active, and male; and sulfur, which was watery, passive, and female. By the sixteenth century, the dominant belief was that metals were composed of three components; for example, Paracelsus (1493-1541) proposed a "*tria prima*," of mercury, sulfur, and salt, which he held to underlie all mater.

[Insert Table 3 about here.]

The egg. The egg was used widely in analogies. Taken as a whole, the egg could symbolize the limitlessness generativity of the universe. Thus the Philosopher's Stone was often called an egg (Cavendish, 1967; Stillman, 1924). The egg could also be divided into components. For example, Stillman (1924) notes that the shell, skin, white, and yolk of the egg were thought to be analogous to the four metals involved in transmutation--copper, tin, lead, and iron--although the pairings could vary between the components and the metals. Several additional correspondences are apparent in the following passage, copied in 1478. In this excerpt, translated from Bertholet's (1887) *Collection des anciens alchemistes grecs*, the "egg" described is in fact the Philosopher's Stone:[13]

Nomenclature of the Egg. This is the mystery of the art.

1. It has been said that the egg is composed of the four elements, because it is the image of the world and contains in itself the four elements. It is called also the "stone which causes the moon to turn," "stone which is not a stone," "stone of the eagle," and "brain of alabaster."

2. The shell of the egg is an element like earth, cold and dry; it has been called copper, iron, tin, lead. The white of the egg is the water divine, the yellow of the egg is couperose [sulfate], the oily portion is fire.

3. The egg has been called the seed and its shell the skin; its white and its yellow the flesh, its oily part, the soul, its aqueous, the breath or the air. (Stillman, 1924, pp. 170-171; notation in brackets added)

This brief excerpt illustrates the style of analogizing displayed by many alchemists. First, the egg is compared to several different analogues. The use of multiple analogues would not in itself differentiate this passage from the work of Boyle; however, there are some differences. First, there does not appear to be a common abstraction across the different analogues. The first paragraph maps the egg first onto the four elements and then onto a series of single entities (e.g., "the stone which is not a stone," the "brain of alabaster"). In (2) and (3), the components of the egg are successively compared to the four elements of ancient Greek philosophy (earth, water, air, and fire),[14] the layers of a seed, and the aspects of a human being. These multiple analogies are rather different from those of Boyle, in part because the alchemist does not attempt to delineate a common structure that holds across the several systems.

A more striking difference from Boyle arises when we consider the issue of one-to-one mappings. (It will be recalled that one-to-one correspondence is one of the constraints in current analogizing, and that Carnot and Boyle both honored this principle.) Figure 4 shows the object correspondences in the above set of analogies. It is apparent that achieving one-to-one correspondence is not of primary concern. Indeed, the number of components involved in the correspondence varies from analogue to analogue. For example, as Figure 4a shows, the object correspondences for the analogy between the egg and the four elements of matter are such that the element of air must be either omitted (hard to imagine, since it is clearly one of the four elements of matter) or else placed in correspondence with a previously used element of the egg, yielding a mapping of four objects onto five. As Figure 4b shows, the mapping from the egg to the four divisions of the seed (or aspects of a human being) is also not one-to-one, since both the white and the yellow parts of the egg correspond to the flesh. Thus Figure 4b shows a $5 \rightarrow 4$ mapping, whereas Figure 4a shows a $4 \rightarrow 5$ mapping.

[Insert Figures 4a and 4b about here.]

An attractive aspect of the egg was that it was recognized as something vital and as symbolic of a beginning. Any system that could be related to the egg was imbued with a similar significance. When some alchemists shifted from the ancient Greek theory of four elements to the theory that three "principles"--usually defined as sulfur, mercury and salt (Cavendish, 1967)--composed all matter, at least one alchemist (for whom arsenic supplanted salt) continued to find the egg analogy appealing:

> As an egg is composed of three things, the shell, the white, and the yolk, so is our Philosophical Egg composed of a body, soul, and spirit. Yet in truth it is but one thing [one mercurial genus], a trinity in unity and unity in trinity--Sulphur, Mercury, and Arsenic. (Dienheim, in Hamilton-Jones (Ed.), 1960, p. 79; brackets are his)

In this passage the alchemist Dienheim suggests a series of parallel analogies among the egg, the Philosopher's Stone, humankind, and matter and gives the object correspondences among the (now three) parts of the egg, the three aspects of a human being, and the three principles of matter. However, he stops short of describing the commonalities that follow from these object correspondences. This passage illustrates the macrocosm-microcosm analogy in alchemical thought and the importance of parallels between the material and spiritual planes. It also illustrates the elusiveness of alchemical analogy: There is no commitment to finding a common abstraction.

Paracelsus. As a further example of the use of analogy in alchemical writing, we present this passage from Paracelsus (1493-1541). Paracelsus (Theophrastus Bombastus von Hohenheim) was a leading alchemist of the sixteenth century and a strong proponent of the value of empirical observation as opposed to received dogma. But despite this pioneering spirit, his use of the analogy remains distinctly different from modern usage. Here, he describes how gold and silver can be made:

> Some one may ask, what, then, is the short and easy way whereby Sol and Luna may be made? The answer is this: After you have made heaven, or the sphere of Saturn, with its life to run over the earth, place on it all the planets so that the portion of Luna may be the smallest. Let all run until heaven or Saturn has entirely disappeared. Then all those planets will remain dead with their old corruptible bodies, having meanwhile obtained another new, perfect and incorruptible body. That body is the spirit of heaven. From it these planets again receive a body and life and live as before. Take this body from the life and earth. Keep it. It is Sol and Luna. Here you have the Art, clear and entire. If you do not understand it it is well. It is better that it should be kept concealed and not made public. (Quoted in Jaffe, 1976, p. 23)

Here Sol and Luna (the sun and moon, respectively) signify gold and silver, and other metals in the recipe are represented by the other planets, according to a widely used system of alchemical analogies (see below). Paracelsus does not detail the object correspondences between the two domains, nor does he explain how an action in one domain parallels an action in the other. The mappings and the theoretical basis for the procedure are left unstated. Indeed, the actual metals being referred to are not always clear. For example, to what do "earth" and "all those planets" refer? Does "heaven, or the sphere of Saturn" refer to tin? If so, is the final "spirit of heaven" derived from the process also tin? This last seems implausible, since the goal is to produce gold and/or silver; yet if the final "spirit of heaven" is gold or silver, then what about the initial "heaven"?

This passage, though it exemplifies the different rules of analogizing among the alchemists, also raises questions concerning the reasons for these differences. Paracelsus makes it clear in the last sentence that clarity is not his intention. The secretive nature of the enterprise, the fact that it was felt necessary to hide results from the common public and perhaps from competitors, perhaps led to the ambiguity of the writing. Is it possible that this ambiguity shielded a set of informative analogies? To answer this question, we must look more closely at the system of analogies that supported this reasoning.

The system of correspondences. Metals held an important place in alchemical analogies. As discussed above, metals figured in analogies with the principles of matter and with the component parts of a human being, and the transmutation of base metals into gold or silver was felt to be analogous to the spiritual purification of man. A further set of rich analogies existed between metals, planets, and colors. The system of correspondences is given in Table 4. (This table and much of the surrounding explication are based on Cavendish's valuable discussion [Cavendish, 1967, p. 26].)

[Insert Table 4 about here.]

The perceived importance of surface similarity is evident here. For example, the Sun, the metal Gold, and the color Gold are linked by a common color, as are the Moon, the metal Silver, and the color White. A second aspect of this set of correspondences is that the commonalities shift from one part of the system to another. For example, unlike the two triads just mentioned, the Jupiter/Tin/Blue triad does not share a common color. Instead, Blue, the color of royalty, is matched to Jupiter because

Jupiter was lord of the sky. The match between Jupiter and Tin may be a color match, based on the planet's silvery appearance. Thus not only are surface similarities implicated, but the decision as to which *particular* surface similarities figure in the correspondences changes from one part of the system to another. A further point of difference between this system and modern systems of analogies is that cross-connections of all kinds enter into the analogies. This excerpt from Cavendish's discussion illustrates the complex web of similarities that underlies the analogies.

> Lead, the darkest and heaviest of the metals, was naturally assigned to Saturn, the dimmest and slowest-moving planet, which trudges heavily through its slow path round the sun. In the old cosmology Saturn is the farthest planet from the sun, the ruler of life, and is the lord of death. The analogy between death and night was drawn very early. Black is the colour of night and the colour invariably associated with death in Western countries. (Cavendish, 1967, p. 27)

As before, there is a marked emphasis on similarity in object attributes, notably color, in determining the correspondences. For instance, Black, Lead, and Saturn are all linked through the surface attribute "dark." A second example of this emphasis on relatively low-order information is the fact that Lead and Saturn were held to match because both are slow and heavy. In fact, the relation between slowness and heaviness is different for the two domains. Saturn moves slowly in its orbit and was therefore thought of as massive ("heavy"). In contrast, lead was known to be a dense ("heavy") metal. Thus the two senses of heaviness (i.e., *large and massive* vs. *dense*) matched here are not the same. Moreover, the direction of inference is different for the two domains: Lead is heavy and therefore inferred to be slow; Saturn is slow and therefore inferred to be heavy. The looseness of the matches between heaviness and slowness in the two domains did not apparently count against the analogy.

Still another difference from modern usage that stands out here is the extreme variety in the types of relations that could justify a given object correspondence. For example, consider the connection between Saturn and Black. Saturn is the lord of death; death is (in some ways) similar to night, and the color of a night sky is Black; further, Blackness symbolizes death. Thus at least two chains exist between the planet Saturn and the color Black.

The heterogeneity of matches that could figure in an analogy here contrasts sharply with the modern aesthetic in which only relations that are parallel across the domains count for the analogy.[15] In a modern analogy we would expect identical relations to hold across the system; that is, we would expect to find the *same* relations holding for each pair:

> Moon:White :: Sun:Golden :: Jupiter:Blue :: Saturn:Black

In the alchemical system there is no such requirement: The relations that link Jupiter and Blue are allowed to be completely different from those that link Moon and White.[16] As another instance of relational heterogeneity, consider the match between Red and Mars. Cavendish (1967, p. 27) notes that it is based on several chains of associations: (a) Mars looks Red; (b) Mars was the god of war, war is associated with bloodshed, and blood is Red; (c) faces are painted Red in war; (d) Mars is held to rule violent energy and activity, and Red is the color symbolizing energy. Because of these multiple paths, Mars and Red were held to be analogous. This illustrates how alchemists differ from modern analogizers with respect to the "no extraneous relations" rule. In the current aesthetic, once the parallel set of relations is established, other relations do not add to the analogy. But for the alchemists, finding more connections improved the correspondence.

Discussion

The alchemists' use of analogy in their writings differed from that of Boyle and Carnot and other more modern scientists. In the examples we have considered, it can be seen that the alchemists violated almost every one of six precepts for analogical rigor given in Table 1 and recapitulated here:

1. Structural consistency is enforced: Objects are placed in one-to-one correspondence, and predicate connectivity (or *support*) is maintained.
2. Relational systems are preserved and object descriptions disregarded.
3. Systematicity is used to select the most informative common relational network.
4. Between-domain relations do not strengthen an analogy.
5. Mixed analogies are avoided.
6. Analogy is not causation.

These disparities seem to represent a true difference in the style of analogical reasoning. Yet before drawing conclusions, we must consider two other factors that may have contributed to the differences. First, the vagueness inherent in alchemical analogy might have stemmed from a desire for secrecy, as discussed above. Certainly the desire for secrecy played a role in the ambiguous quality of alchemical analogy. In order to prevent laypeople from understanding the mysteries of alchemy, its practitioners disguised their recipes with symbolism and vagueness, and this undoubtedly contributed to the ambiguity of the analogies. But although this explanation is probably correct as far as it goes, it will not account for all of the facts. In particular, it will not account for the alchemists' fondness for correspondences based on (a) surface similarity and (b) multiple linking paths, for it is precisely these kinds of correspondences that would easily be guessed by an outsider. For example, the connection among the Moon, the metal Silver, and the color White would have been easy for an outsider to deduce; and the rich set of relations linking Mars and Red made it unmistakable that the two should be placed in correspondence. In modern analogy, the object correspondences are often more difficult to grasp *initially* than in alchemical analogies because the correspondences are based purely on like roles in the matching relational system, with no direct object similarity. For example, in Boyle's analogy between ants moving a mass of eggs and wind stripping the leaves off a tree, the object correspondences between ants and air particles and ant eggs and leaves are not at all obvious a priori; they are not suggested by surface similarity, nor are there multiple paths linking, for example, air particles and leaves. Thus a modern analogy may be far harder for a newcomer to grasp initially than are the alchemists' analogies. Clearly, not all the disparities between alchemical analogy and modern analogy can be accounted for by the desire to achieve secrecy.[17]

A second and deeper difference between alchemists and modern scientists is the fact that the alchemists had rather more complex goals. They were concerned with understanding both the material and spiritual worlds, and they used several forms of macrocosm-microcosm analogies to link the two planes. Alchemists often invested this analogy between the spiritual and material planes with dual-causal powers. A scientist who wished to purify a base metal into gold must, they thought, also purify his spirit. Modern science separates personal virtue from excellence in research, and although this separation has its disadvantages, it simplifies the enterprise. To compound this difference in goals, it has been suggested that the alchemists may have been relatively more focused on power and control than on knowledge. It is hard to say how much of the apparent disparity in reasoning style might have stemmed from these different motivations.

With the foregoing cautions, we now consider whether the disparities in analogizing suggest a genuine difference in reasoning style. (See also Campbell, 1987, for a discussion of factors relating to such a conclusion.) Some of the differences--notably violations of precepts 2 (preserve relations rather than attributes) and 3 (aim for systematicity); see *The rules of analogical rigor*, above--could reasonably be attributed to simple lack of domain knowledge. Later scientists, such as Carnot and Boyle, had the benefit of more extensive sets of existing principles on which to base their analogies. The alchemists' use of surface similarity instead of common relational structure could be defended as a perfectly reasonable initial way to proceed, given the relative lack of domain knowledge. Indeed, there is considerable evidence from studies of analogical development (Billow, 1975; Gentner, 1989; Gentner & Toupin, 1986) and from novice-expert studies in learning physics (Chi, Feltovich, & Glaser, 1981) to suggest that novice learners judge similarity by common object attributes whereas adults judge similarity by common relational structure. Such a bias can be defended on grounds of cognitive economy: Why postulate relational commonalities until you are sure that attribute commonalities are inadequate? Thus the alchemists' deviations on precepts 2 and 3 cannot be taken as evidence of a different style of thinking, only of a difference in amount of knowledge.

When we turn to the remaining precepts, the domain knowledge interpretation is less plausible. The fact that the alchemists felt no need for one-to-one correspondences, their fondness for between-domain relations and mixed analogies, and their propensity to ascribe causal powers to analogy and similarity all seem to point to a true difference in their sense of the implicit rules of analogy. Thus the alchemists, in attempting to gain an understanding of their world, used a very different set of inference rules from that of later scientists. Returning to the central question of this chapter, we conclude that the rules of analogical soundness are not innate. Despite the seeming inevitability of the analogical precepts we now use, they are not a necessary part of natural logic.

The style of analogical reasoning in alchemy and chemistry seems to have changed between the time of the Paracelsus and that of Boyle (1627-1691). This change was to some degree domain-specific, for true analogies were used in physics and astronomy before they were in alchemy and chemistry. Kepler (1571-1630) and Galileo (1564-1642), each working within about 70 years of Paracelsus, were as elegant in their use of analogy as any modern thinker. For example, Kepler, grappling with the notion of action at a distance, developed a deep analogy between light and a force he hypothesized to emanate from the sun. Just as light cannot be apprehended as it travels through the space, yet produces an effect when it reaches its destination, so might it be with this new force.[18] Galileo used an analogy between the earth and a ship to argue that the earth moves despite the evidence of our senses (see Gentner, 1982). These analogies are as rigorous and systematic as the analogies of modern scientists. This makes the contrast in analogical style between, say, Paracelsus and the later chemists all the more striking. It suggests a domain-specific progression in alchemy and chemistry from one set of implicit rules governing the practice of analogical reasoning in 1500 to another set in 1700. (Whether a similar evolution occurred in astronomy and physics prior to 1600 and whether the practice in alchemy was influenced by the more rigorous practice in physics and astronomy are issues beyond the scope of this report.)

The evidence reviewed here suggests that analogical rigor as we practice it today has not been universal in the history of science. The skilled practice of analogical reasoning does not appear to be an innate human skill, and learning the habit of rigorous analogizing does not appear to be a universal achievement like learning the grammar of a language. Yet we do not wish to take the opposite position, that analogy is an esoteric ability available only to a few. On the contrary, we suspect that the ability to see relational matches at least some of the time is universal. What does not appear to be universal is a demarcation between analogy and other forms of similarity, in which a special role and a distinct set of rules are accorded to analogy in reasoning.

Perhaps analogy is more like mathematics than it is like language. If we liken the human intuitive perception of similarity to our intuitive ability to estimate numerosity, then possessing the rules of analogical rigor is like possessing the rules of arithmetic. The analogy can be pursued further. Just as whole cultures existed and estimated quantities without inventing key notions of arithmetic (such as the idea of a zero), so a people may use similarity comparisons without developing the notion of a sound analogy. Again, in a premathematical society, instances of perfectly correct calculation will occur intermixed with other less reliable kinds of estimation. So too with analogy: For example, some of the alchemists' comparisons would qualify as sound analogies, though many would not. But the most important commonality is that once a rigorous method has been culturally codified, it is accorded a special role. Strict analogy, like arithmetic, is now the method of choice when correctness is important.

Finally, in neither case do the formal methods totally supplant the prior forms of reasoning. There are occasions when rough estimation is more appropriate than carrying out arithmetic; and there are occasions--such as reading poetry--when appearance matches or mixed metaphors are more appropriate than strict analogy.

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Authors' Notes

We thank Cathy Clement, Brian Falkenhainer, Ken Forbus, Monica Olmstead, Mary Jo Rattermann, Bob Schumacher, and Janice Skorstad for discussions of these issues, and for comments on prior drafts of this report. This report also appears as a chapter in B. Gholson, W. R. Shadish, Jr., R. A. Neimeyer, & A. C. Houts (Eds.), *Psychology of science: Contributions to meta-science*.

Footnotes

1. We originally intended to use models of heat as a unifying theme, and indeed the passages from Boyle and Carnot are both concerned in part with the nature of heat. However, we were not successful in finding alchemical passages dealing extensively with heat, and so the alchemical passages considered here cover a range of phenomena.

2. The order shown here should not be taken as the order of processing; in fact, selecting the object correspondences may often be the last step (Falkenhainer, Forbus, & Gentner, 1986, 1989/90).

3. The order of a relation is determined by the order of its arguments. A first-order relation is one that takes objects as its arguments. A second-order relation has at least one first-order relation among its arguments. An nth-order relation has at least one (n - 1)th-order argument.

4. Systematicity is operationalized in the computer simulation of structure mapping as follows: Any match between two relations in base and target--for example, MORE MASSIVE THAN (sun, planet) and MORE MASSIVE THAN (nucleus, electron)--is given a higher evaluation if the parent relation (i.e., the relation immediately dominating them) also matches (Falkenhainer, Forbus, & Gentner, 1986, 1989/90; Gentner, 1989).

5. As with the other precepts, there are occasional violations of this maxim: For example, in a survey of the analogies used to explain cognition in the history of psychology, Gentner and Grudin (1985) found that certain brain-based analogies (such as "concepts as reverberating circuits") seemed to take on extra authority because of the known causal connection between brain and cognition.

6. The caloric theory was widely accepted until Joule and other experimenters in the 1840s demonstrated the interconvertability of heat and work (Wilson, 1981). Carnot's reliance on the caloric theory did not invalidate his basic conclusions regarding the cycle, although some later statements in *Reflexions* are unsound when viewed from the perspective of the mechanical theory of heat (Fox, 1971).

7. It has been suggested that Carnot's theories were strongly influenced by the work of engineers of his era, and that his book was intended to advance engineering technology (Cardwell, 1965; Fox, 1971; Kuhn, 1959) and popularize the use of heat power (Wilson, 1981). This purpose would explain Carnot's need for the analogy as an explanatory device.

8. Although Carnot refers to a waterfall, his discussion may have been based not merely on waterfalls, but on some kind of water engine, such as a water wheel or a column-of-water engine (Cardwell, 1965).

9. Carnot's solution to this question was affected by his reliance on the questionable data of other scientists. For a detailed discussion see Fox (1971). For our purposes, however, the answer to the question is not as important as the fact that the question arises from the analogy.

10. It is tempting to speculate, along the lines of Hesse's (1966) insightful discussion, that at least part of the difference in analogical style between Carnot and Boyle stems from differences in intellectual tradition among French and English. Hesse notes that French academics were inclined to think of analogy as vague and unsatisfactory, at best a mental crutch to use until a formal model could be devised. In contrast, in the English tradition, mechanical analogies were valued as sources of insight, especially with respect to preserving causation. From this perspective it is not surprising that Boyle is a more enthusiastic analogizer than Carnot.

11. This discussion is taken largely from Cavendish (1967, pp. 143-180).

12. Boyle, in the seventeenth century, was among the first to challenge this doctrine.

13. Although this passage was copied in 1478, its exact date of origin is difficult to pinpoint. Other manuscripts from this collection are believed to have existed since before the fourth century in one form or another (Stillman, 1924).

14. However, this is an unusual (perhaps a transitional) account of the elements. The elements listed are earth (or metal), water, couperose (or sulfur or sulfate) and fire, with air not explicitly mentioned.

15. Contrast the complexity and elusiveness of the relations underlying Table 4, and especially their variability across rows and columns, with the factorial regularity of the relations underlying the modern periodic table of the elements.

16. An alternate way of describing the alchemical aesthetic is to say that the relations involved are extremely nonspecific, for example, "associated in some way" or "often co-occurring." Under that description, the alchemist would not be guilty of shifting relations between parallel analogues. However, this degree of nonspecificity of relations would still constitute a marked difference from modern usage.

17. However, the penchant for secrecy might have had indirect effects if it discouraged group collaboration on the analogies. As Boyd (1979) points out, one striking difference between scientific analogy and literary metaphor as practiced today is that an explanatory analogy is considered to be part of the public domain, so that it is common for scientists to improve on one another's analogies. If nothing else, the alchemical desire for secrecy must have interfered with this process of collegial tinkering.

18. This force is clearly a precursor of Newton's notion of gravity, about 80 years later.

Table 1

Constraints on Analogical Reasoning

1. *Structural consistency.* (a) Objects from base and target are placed in one-to-one correspondence. (b) Predicate connectivity, or *support*, is preserved in the mapping.

2. *Relational focus*. Relational systems are preserved and object descriptions disregarded. Object correspondences are determined not by intrinsic resemblances between the objects but by whether the objects participate in identical systems of relations.

3. *Systematicity*. In selecting among several common relations, common systems of relations are preferred: Lower order relations governed by a higher order relation are more likely to be included in the interpretation of an analogy than are isolated lower order relations.

4. *Between-domain relations do not strengthen an analogy*. Additional connections between the base and target domain do not increase the soundness of a match.

5. *Mixed analogies are avoided.* The relational network to be mapped should be entirely contained within one base domain; it is unsound to combine relations from several base domains.

6. *Analogy is not causation.* An analogical resemblance between two situations is not evidence that one of them causes the other.

Table 2

An Overview of Boyle's Series of Analogies Concerning Local Motion

Abstract model	Layman's view [1]	Analogues [2]	[3]	[5]	[6]	[7]	[8]
Small particles	Dust	Ants	Air particles	Aqueous particles	Igneous particles	Animal spirits	Aqueous corpuscles
Large bodies	Large bodies	Mass of eggs	Tree	Mercury oxide	Metal	Animals	Rope
Fragments of bodies	Fragments of bodies	Single eggs	Leaves	Grains of oxide	Metal corpuscles	Animal [inner parts]	Rope [inner parts]

Table 3

Dienheim's Analogy and Related Analogies of the Later Alchemists

Dienheim's analogy			Further correspondences[a]	
Three parts of the egg	Three components of the Philosopher's Stone	Three elements of matter	Two male-female principles	Four primary qualities
White	Soul	Sulfur	Male	Fire
Yolk	Spirit	Mercury	Male-female	Air/water
Shell	Body	Arsenic (salt)	Female	Earth

[a]Most of these correspondences were in common use during later alchemical times.
Source: Columns 2-5 are from Cavendish (1967, p. 169).

Table 4

The Alchemical System of Correspondences Among Planets, Metals, and Colors

Planets	Metals	Colors
Sun	Gold	Gold, yellow
Moon	Silver	White
Mercury	Quicksilver	Gray, neutral
Venus	Copper	Green
Mars	Iron	Red
Jupiter	Tin	Blue
Saturn	Lead	Black

Source: Cavendish (1967, p. 26).

Figure Captions

Figure 1. Carnot's analogy: the common relational structure for water and heat.

Figure 2. Propositions derivable from Carnot's water/heat analogy.

Figure 3. Boyle's analogy: the common relational structure for ants moving eggs and wind blowing leaves.

Figure 4. Object correspondences in the egg analogy.

Figure 1

Carnot's Analogy: The Common Relational Structure for Water and Heat

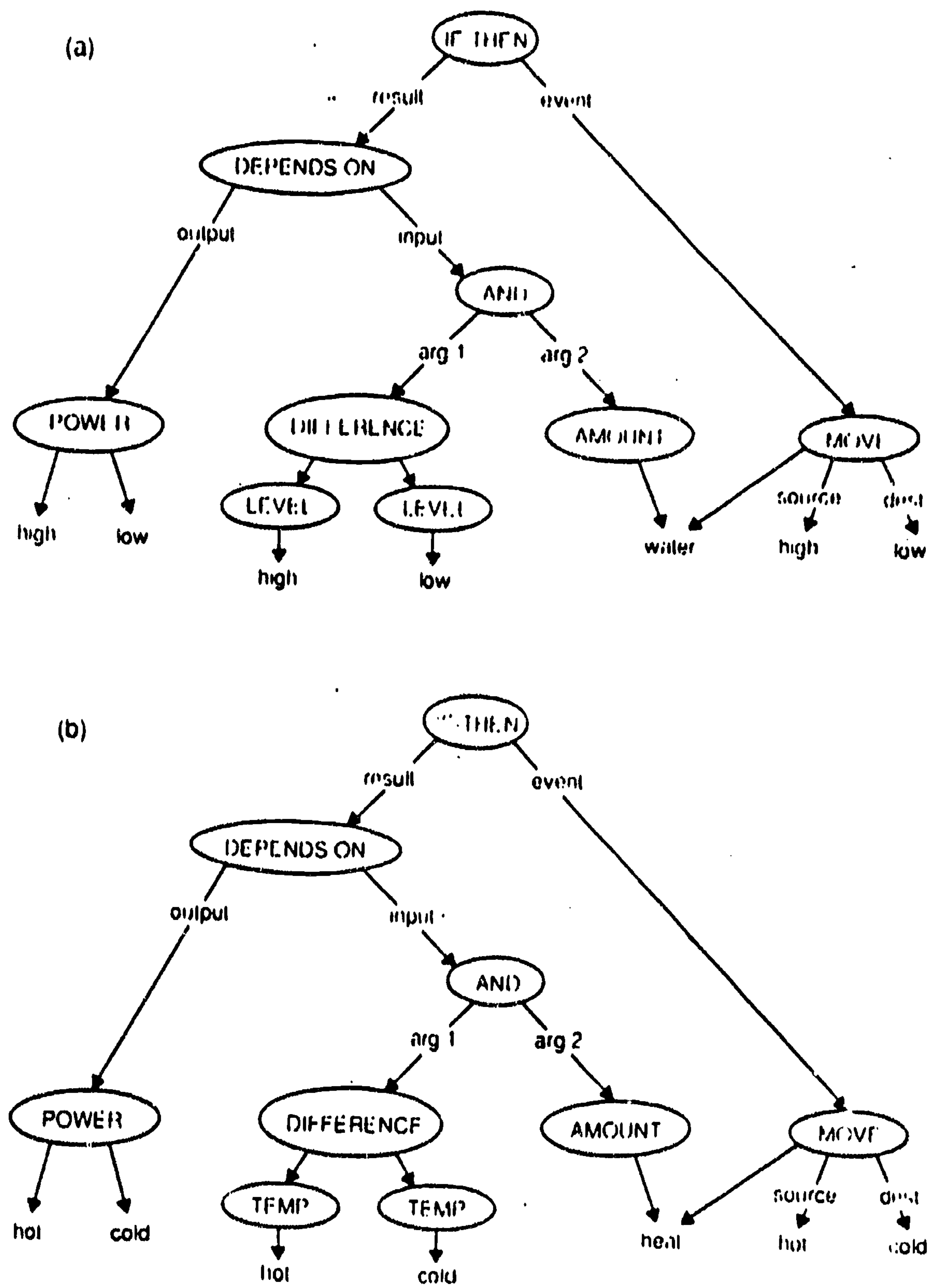


Figure 2

Propositions Derivable From Carnot's Water/Heat Analogy

1. * water: DIFFERENCE (level <h>, level <l>)
   * heat: DIFFERENCE (temp <h>, temp <c>)

2. * water: FLOW (h, l)
   * heat FLOW (h, c)

3. * water POWER (h, l)
   * heat POWER (h, c)

4. * water: MAX POWER (h, l)
   * heat: MAX POWER (h, c)

5. * water: $\propto_Q$ [POWER (h, l), DIFFERENCE (level <h>, level <l>)]
   * heat: $\propto_Q$ [POWER (h, c), DIFFERENCE (temp <h>, temp <c>)]

6. * water: $\propto_Q$ [POWER (h, l), amt <h>]
   * heat: $\propto_Q$ [POWER (h, c), amt <h>]

7. * water: AND [$\propto_Q$ [POWER (h, l), DIFFERENCE (level <h>, level <l>)],
   $\propto_Q$ [POWER (h, l), amt <h>]]
   * heat: AND [$\propto_Q$ [POWER (h, c), DIFFERENCE (temp <h>, temp <c>)],
   $\propto_Q$ [POWER (h, c), amt <h>]]

8. * water: CAUSE [DIFFERENCE (level <h>, level <l>), FLOW (h, l)]
   * heat: CAUSE [DIFFERENCE (temp <h>, temp <c>), FLOW (h, c)]

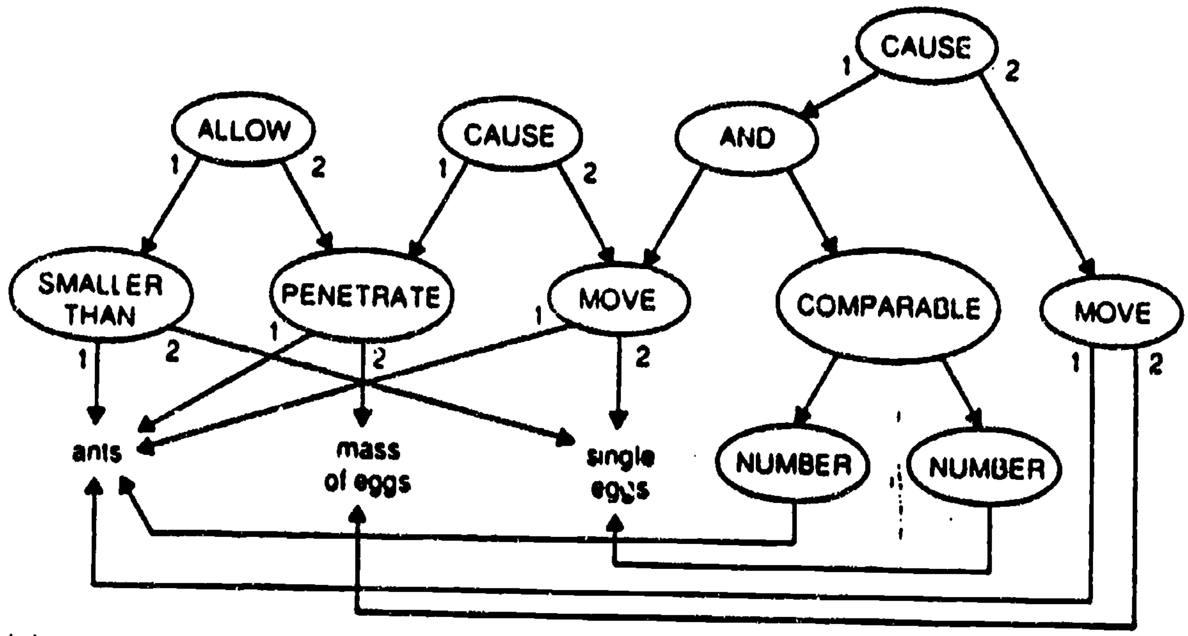
9. * water: $\propto_Q$ [FLOW (h, l), DIFFERENCE (level <h>, level <l>)]
   * heat: $\propto_Q$ [FLOW (h, c), DIFFERENCE (temp <h>, temp <c>)]

10. * water: CAUSE [DIFFERENCE (level <h>, level <l>, POWER (h, l)]
    * heat. CAUSE [DIFFERENCE (temp <h>, temp <c>, POWER (h, c)]

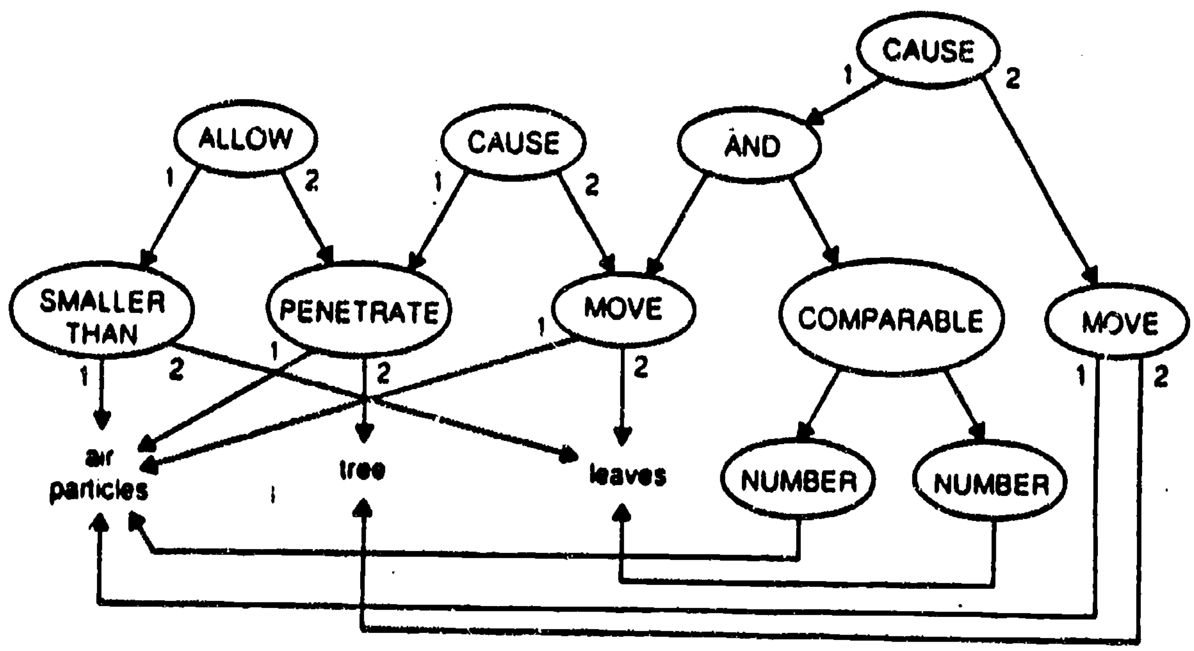
11. * water: CAUSE [FLOW (h, l), POWER (h, l)]
    * heat: CAUSE [FLOW (h, c), POWER (h, c)]

Figure 3

Boyle's Analogy: The Common Relational Structure for Ants Moving Eggs and Wind Blowing Leaves



(a)



(b)

Figure 4

Object Correspondences in the Egg Analogy

a)

EGG		FOUR ELEMENTS
shell	→	earth (copper, iron, tin, lead)
white	→	water
yellow	→	couperose (sulfur?)
oily portion	→	fire
		(air?)

b)

EGG		SEED
shell	→	skin
white	→	flesh
yellow	→	flesh
oily portion	→	soul
aqueous	→	breath/air